

$\eta' - g^* - g$ Transition Form Factor with Gluon Content Contribution Tested

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Abstract

We study the $\eta' - g^* - g$ transition form factor by using the η' wave function constrained by the experimental data on the $\eta' - \gamma^* - \gamma$ transition form factor provided by CLEO and L3 . We also take into account the contribution of the possible gluonic content of the η' meson.

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The large branching ratios of $B \rightarrow \eta' X_s$ and $B \rightarrow \eta' K$ detected by CLEO [1,2] enhanced the importance of the QCD transition form factor $F_{\eta' g^* g} \equiv H(q_1^2, q_1^2, m_{\eta'}^2)$. The mechanisms using this transition form factor to explain these large decay rates are based on the $b \rightarrow sg^*$ penguin transition followed by the decay $g^* \rightarrow \eta' g$ [3–8] or on the transitions $g^* g^*$, $g^* g \rightarrow \eta'$ [9,10]. The transition form factor $F_{\eta' g^* g}$ being used are either extracted from the experimental data of $\psi \rightarrow \eta' \gamma$ [3,9,10], or based on phenomenological consideration [4,8], or calculated by assuming pseudoscalar coupling between η' and a quark pair [5]. Thus a question arises, what is the relation between the form factor $F_{\eta' g^* g}$ used in the above references and the wave function of η' ? Provided that the form factors $F_{\eta' g^* g}$ used in these references are not the same, which one can be got from the calculation based on the structure of η' ? Thus studying the QCD transition form factor $F_{\eta' g^* g}$ is not only important in investigating the dynamics of η' production from B decays, but also in detecting the structure of η' .

The η' meson is particularly different from the flavor octet meson: π , K . It is mainly a singlet meson. According to the QCD anomaly it is much heavier than the massless Glodstone boson [11]. Because of its singlet structure the η' meson may have gluonic content. Since two decades ago its gluonic structure has been studied in QCD sum rules [12]. Recently the experimental data on the $\eta' \gamma^* \gamma$ transition form factor from CLEO [13] and L3 [14] pushed forward the development of the investigations of the quark structure of η' [15]. A new $q\bar{q} - s\bar{s}$ (here $q\bar{q}$ means $u\bar{u}$ and $d\bar{d}$) mixing scheme was developed [16]. The calculation based on the η' wave function successfully describes the experimental data on the $\eta' \gamma^* \gamma$ transition form factor over a wide range of the virtual photon momentum squared, $1\text{GeV}^2 \leq Q^2 \leq 15\text{GeV}^2$. Here the momentum squared of the virtual gluon in the production of η' from B decay can vary from 1GeV^2 to 25GeV^2 . Is the situation in $g^* g \rightarrow \eta'$ or $g^* \rightarrow \eta' g$ transition similar to $\gamma^* \gamma \rightarrow \eta'$ transition? Maybe not because of the particular QCD structure of η' . The QCD anomaly and gluonic content in the η' may play important role in the $\eta' - g^* - g$ transition. The QCD axial anomaly determines the behavior of the $\eta' - g^* - g$ transition form factor at small momentum transfer, i.e., in the soft limit $Q^2 \rightarrow 0$. In the range $Q^2 \geq 1\text{GeV}^2$ gluonic content of η' may have some contributions, which will make difference between the $\eta' - g^* - g$ transition and $\eta' - \gamma^* - \gamma$ transition.

In this work we use the wave function of η' to calculate the $\eta' - g^* - g$ transition form factor $F_{\eta' g^* g}$ at the large momentum transfer region $Q^2 \geq 1\text{GeV}^2$. We not only take into account the quark content of η' but also test how much the gluonic content contributes.

In the $\frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle$ and $|s\bar{s}\rangle$ mixing scheme [16] the parton Fock state decomposition can be expressed as

$$|\eta'\rangle = \sin\phi |\eta'_q\rangle + \cos\phi |\eta'_s\rangle + |G\rangle. \quad (1)$$

where ϕ is the mixing angle, and $|\eta'_q\rangle \sim \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle$, $|\eta'_s\rangle \sim |s\bar{s}\rangle$, $|G\rangle \sim |gg\rangle$. In eq.(1) $|\eta'_q\rangle$ and $|\eta'_s\rangle$ are quark Fock states, and $|G\rangle$ is the gluonic Fock state.

$|G\rangle$ and $|\eta'_q\rangle, |\eta'_s\rangle$ are not independent. Because $|\eta'_q\rangle$ and $|\eta'_s\rangle$ are non-flavor-octet, the evolution of their wave function will mix with the two-gluon state. In [17] the evolution equation for the wave functions of the mixing $q\bar{q}$ and $g\bar{g}$ state has been derived. The eigenfunctions have been calculated. After a few simple steps of algebraic procedures, one finds,

$$\begin{aligned}\Psi_i^q(\mu^2, x) &= f_i \phi^q(\mu^2, x), \\ \phi^q(\mu^2, x) &= 6x(1-x) \left\{ 1 + \sum_{n=2,4,\dots} [B_n^q \left(\frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)} \right)^{\gamma_+^n} + \rho_n^g B_n^g \left(\frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)} \right)^{\gamma_-^n}] C_n^{3/2}(2x-1) \right\} \\ \Psi_i^g(\mu^2, x) &= f_i \phi^g(\mu^2, x), \\ \phi^g(\mu^2, x) &= x(1-x) \left\{ \sum_{n=2,4,\dots} [\rho_n^q B_n^q \left(\frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)} \right)^{\gamma_+^n} + B_n^g \left(\frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)} \right)^{\gamma_-^n}] C_n^{5/2}(2x-1) \right\}, \quad (2)\end{aligned}$$

where the superscripts q and g indicate the “quark” and “gluon” content, i denotes the meson state composed of the $q\bar{q}$ pair, and f_i is the decay constant of the relevant meson state of $q\bar{q}$. The parameter x is the momentum fraction carried by the parton. Here μ is the scale of the hard process, which may be taken to be the momentum transfer Q^2 involved in the hard process. $\gamma_+^n, \gamma_-^n, \rho_n^g, \rho_n^q$ are not free parameters; they are given in Appendix A. $C_n^{3/2}$ and $C_n^{5/2}$ are Gegenbauer polynomials. μ_0 is the reference scale and we take $\mu_0 = 0.5\text{GeV}$ in our calculation. Because of the general symmetry properties of the wave functions of the two-particle bound state of a neutral pseudoscalar meson, the quark wave function satisfies $\phi^q(x) = \phi^q(1-x)$, and for the gluon function $\phi^g(x) = -\phi^g(1-x)$.

The diagrams of the $\eta' - g^* - g$ transition amplitude are shown in Fig.1 and Fig.2. Note that we do not include the diagrams with s-channel gluon exchange between the g^*g and $q\bar{q}$ (gg) here, because they are color suppressed in the case that $q\bar{q}$ (gg) will hadronize into mesons. According to the symmetry properties of the gluon wave function, it is easily found that Fig.2(c) does not contribute to this amplitude.

The contribution of the quark wave function to the $\eta' - g^* - g$ vertex (see Fig.1) is

$$\begin{aligned}T^q &= c \int_0^1 dx \frac{1}{4N} \phi^q(Q^2, x) \text{Tr}[\gamma_5 \not{p} T_H^q], \\ c &= c_q f_q \sin\phi + c_s f_s \cos\phi,\end{aligned} \quad (3)$$

where N is the color number, T_H^q is the hard amplitude of the quark parton contribution. $c_q = \sqrt{2}$, $c_s = 1$, f_q and f_s are the decay constant of η'_q and η'_s respectively. The gluon wave function contribution is (Fig.2)

$$T^g = c \int_0^1 dx \frac{1}{4N} \phi^g(Q^2, x) \cdot i \epsilon^{\alpha\beta\rho\sigma} q_\alpha l_\beta (T_H^g)^{\mu\nu}_{\rho\sigma}, \quad (4)$$

here $q = (p+q_2)/Q$, $l = (p-q_2)/Q$, and $Q^2 = q_1^2$. Q^2 can be chosen as the evolution scale of the wave functions.

The $\eta'g^*g$ transition form factor $F_{\eta'g^*g}$ can be defined through

$$T^q + T^g = F_{\eta'g^*g}(q_1^2 = Q^2, q_2^2 = 0, m_{\eta'}^2) \cdot \delta^{ab} \varepsilon^{\alpha\beta\mu\nu} q_{1\alpha} q_{2\beta} \quad (5)$$

The indices a and b are $SU(3)_c$ generator indices. Calculating the Feynman graphs shown in Fig.1 and 2, we can obtain

$$\begin{aligned} & F_{\eta'g^*g}(q_1^2 = Q^2, q_2^2 = 0, m_{\eta'}^2) \\ &= 4\pi\alpha_s(Q^2)(c_q f_q \sin\phi + c_s f_s \cos\phi) \\ & \left\{ \frac{1}{2N} \int_0^1 dx \phi^q(Q^2, x) \left[\frac{1}{(x^2 - x)p^2 + (1 - x)q_1^2} + (x \leftrightarrow (1 - x)) \right] \right. \\ & \left. - \frac{1}{2Q^2} \int_0^1 dx \phi^g(Q^2, x) \left[\frac{(1 + x(1 - x))p^2 - xq_1^2}{(x^2 - x)p^2 + (1 - x)q_1^2} + (x \leftrightarrow (1 - x)) \right] \right\}. \quad (6) \end{aligned}$$

The decay constants f_q , f_s and the mixing angle ϕ have been constrained from the available experimental data, $f_q = (1.07 \pm 0.02)f_\pi$, $f_s = (1.34 \pm 0.06)f_\pi$, $\phi = 39.3^\circ \pm 1.0^\circ$ [16]. The free parameters exist in the evolution functions $\phi^q(Q^2, x)$ and $\phi^g(Q^2, x)$ (see eq.(2)). They are B_n^q and B_n^g , $n = 2, 4, \dots$. The fit to the experimental data of the $\eta' - \gamma$ transition form factor shows that $\phi^q(Q^2, x)$ should not be much different from the asymptotic form $\phi_{AS}(x) = 6x(1 - x)$, i.e., the parameters B_n^q and $\rho_n^g B_n^g$ should be small enough (ρ_n^g are not free parameters, see Appendix A). The parameters γ_+^n and γ_-^n are negative and their absolute values increase with n . Consequently it is a good approximation to consider only the first one or two terms in the expansion of the wave function $\phi^q(Q^2, x)$ and $\phi^g(Q^2, x)$. In this work we only take into account the first term in the expansion of the wave functions. We keep $|B_2^q|$ and $|\rho_2^g B_2^g| < 0.1$ in order to keep the constraint of the experiment data of the $\eta' - \gamma$ transition form factor. Because $\rho_2^g = -0.05$, we take $|B_2^g| < 2.0$.

If we adopt the constraints $|B_2^q| < 0.1$, $|B_2^g| < 2.0$, we find that the contribution of the gluon wave function is very small, it can be neglected. Because the gluonic contribution is so small, the dominant contribution comes from the asymptotic quark wave function. The dependence of the QCD transition form factor $F_{\eta'g^*g}$ on the free parameters B_2^q and B_2^g is extremely weak. As an example, we present the functional form of $F_{\eta'g^*g}$ with $B_2^q = 0$, $B_2^g = 2.0$ in Fig.3. We can see from the figure, after taking into account the gluonic wave function, the total result is not different greatly from the quark wave function contribution.

In Fig.4 we compare our result (solid curve in Fig.4) with what were used in the literatures: i) in [5,9,10] the η' -gluon transition form factor is taken as $\frac{H(0,0,m_{\eta'}^2)}{(q_1^2/m_{\eta'}^2 - 1)}$, where $H(0,0,m_{\eta'}^2)$ is a phenomenological parameter which should be extracted from the branching ratio of $\psi \rightarrow \eta'\gamma$, $H(0,0,m_{\eta'}^2) \approx 1.8\text{GeV}^{-1}$ (see the dashed curve in Fig.4); ii) in [4,8] the form factor is taken as $\sqrt{3}\alpha_s(Q^2)/(\pi f_\pi)$ (see the dot-dashed curve in Fig.4). Our result is very close to $\frac{1.8\text{GeV}^{-1}}{(q^2/m_{\eta'}^2 - 1)}$. If we take $H(0,0,m_{\eta'}^2) = 1.7\text{GeV}^{-1}$, the curve is even closer to our result. In [3,5,9,10] the branching fraction

of $\psi \rightarrow \eta' \gamma$ is needed to extract the parameter $H(0, 0, m_{\eta'}^2)$; here we do not need to extract the parameter $H(0, 0, m_{\eta'}^2)$. The structure of η' can determine the behavior of the form factor $F_{\eta' g^* g}$ completely. It is also possible to calculate $F_{\eta' g^* g^*}$ by using the same method. Certainly a large number of experiments such as $pp \rightarrow \eta' x$, $pp \eta'$, etc., are needed to test the behavior of the η' -gluon transition form factor. The behavior of the η' -gluon transition form factor is not only important in investigating the dynamics of η' production from B decays, but also in determining the structure of η' . Thus such kinds of experiments are urgently needed.

The summary: we calculated the QCD transition form factor $F_{\eta' g^* g}(Q^2, 0, m_{\eta'}^2)$ by using the wave function of η' which is obtained by solving the evolution equation [17]. We included the gluonic wave function in our calculation. We find within the possible free parameter region, the gluonic contribution is small, and the QCD transition form factor $F_{\eta' g^* g}$ does not depend on the free parameters greatly.

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Appendix A

The parameters γ^n and ρ_n in eq.(2):

$$\gamma_{\pm}^n = \frac{1}{2} \{ \gamma_{qq}^n + \gamma_{gg}^n \pm \sqrt{(\gamma_{qq}^n - \gamma_{gg}^n)^2 + 4\gamma_{gq}^n \gamma_{qg}^n} \}, \quad (A1)$$

$$\gamma_{qq}^n = \frac{C_F}{\beta} \left\{ \frac{2}{(n+1)(n+2)} - 1 - 4 \sum_{j=2}^{n+1} \frac{1}{j} \right\}, \quad (A2)$$

where $C_F = \frac{4}{3}$, $\beta = \frac{11N-2n_f}{3}$, N is the number of color, n_f is the number of the active quarks.

$$\gamma_{gq}^n = \frac{n_f}{\beta} \frac{2}{(n+1)(n+2)}, \quad (A3)$$

$$\gamma_{qg}^n = \frac{C_F}{\beta} \frac{n(n+3)}{(n+1)(n+2)}, \quad (A4)$$

$$\gamma_{gg}^n = \frac{4N}{\beta} \left\{ \frac{2}{(n+1)(n+2)} - \sum_{j=2}^{n+1} \frac{1}{j} - \frac{1}{12} - \frac{n_f}{6N} \right\}, \quad (A5)$$

where $n \geq 1$.

$$P_n = \frac{\gamma_+^n - \gamma_{qq}^n}{\gamma_+^n - \gamma_-^n}, \quad Q_n = \frac{\gamma_{qg}^n}{\gamma_+^n - \gamma_-^n}, \quad (A5)$$

$$\rho_n^g = -\frac{1}{6} \frac{Q_n}{1 - P_n}, \quad \rho_n^q = 6 \frac{P_n}{Q_n}. \quad (A6)$$

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Figure Captions

Fig1. The quark content contribution to the $\eta' g^* g$ transition

Fig2. The gluonic content contribution to the $\eta' g^* g$ transition

Fig3. The QCD transition form factor $F_{\eta' g^* g}(Q^2, 0, m_{\eta'}^2)$. The dot-dashed curve is the contribution of the quark wave function, the dashed curve is the gluonic contribution ($B_2^2 = 0$, $B_2^g = 2.0$), and the solid one is the total contribution.

Fig4. The comparison of $F_{\eta' g^* g}$ which we obtain with others used in literatures:
i) The solid curve is the result calculated based on the η' wave function; ii) The dashed curve is for $\frac{H(0,0,m_{\eta'}^2)}{(q_1^2/m_{\eta'}^2-1)}$ with $H(0,0,m_{\eta'}^2) = 1.8 GeV^{-1}$; iii) The dot-dashed one is $\sqrt{3}\alpha_s(Q^2)/(\pi f_\pi)$

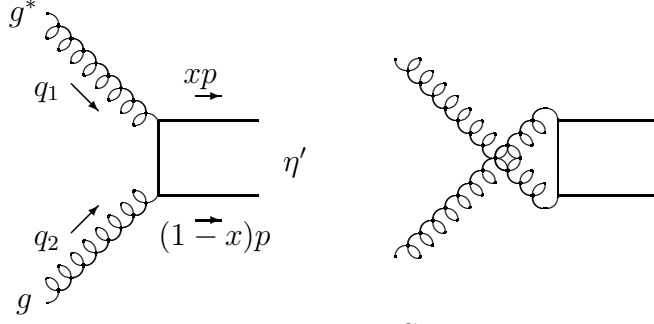


FIG. 1.

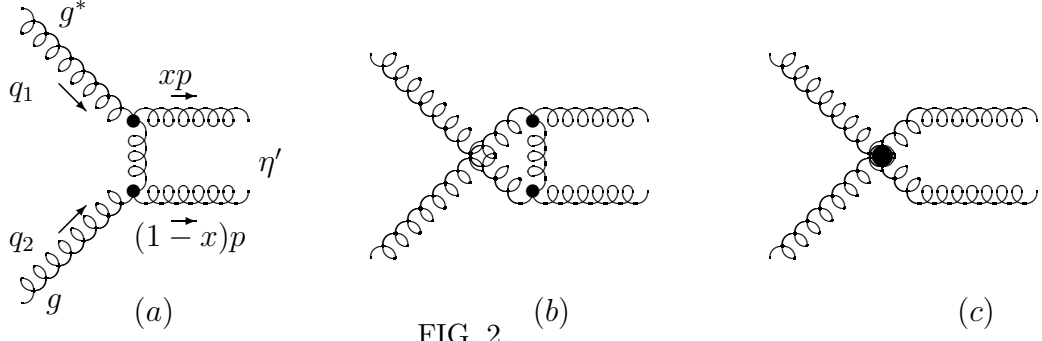


FIG. 2.

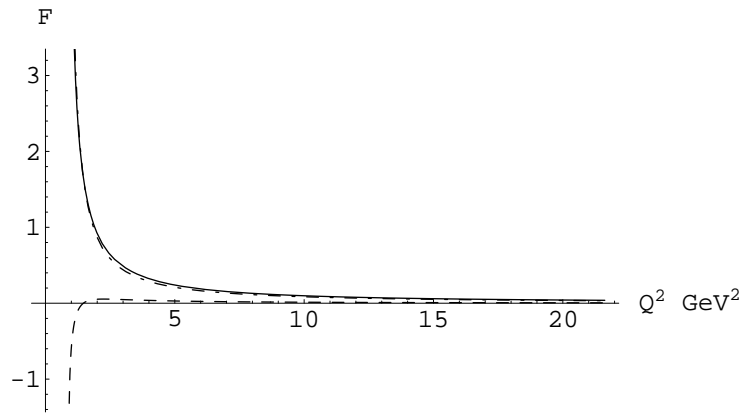


FIG. 3.

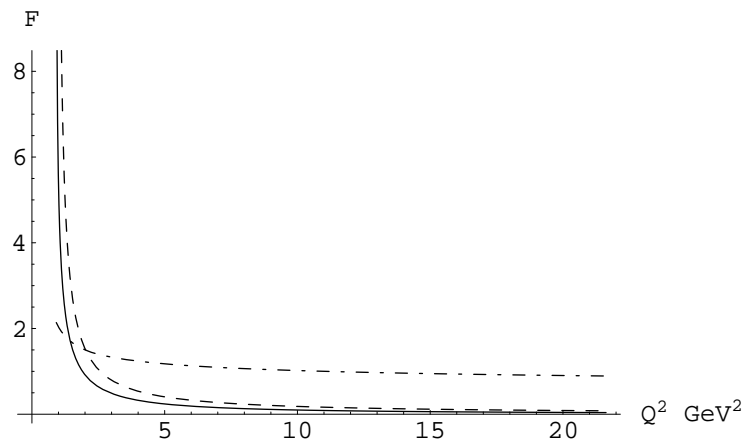


FIG. 4.